# Characterizations of Lie-type derivations on rings and algebras

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**Abstract**

Let $\mathcal{R}$ be a commutative ring with identity, $\mathcal{A}$ be an algebra over $\mathcal{R}$ and $\mathcal{Z}(\mathcal{A})$ be the center of $\mathcal{A}$. An $\mathcal{R}$-linear mapping $L : \mathcal{A} \to \mathcal{A}$ is called a *derivation* if $L(XY) = L(X)Y + XL(Y)$ for all $X, Y \in \mathcal{A}$. Let $[X, Y] = XY - YX$ denote the Lie product of elements $X, Y \in \mathcal{A}$. An $\mathcal{R}$-linear mapping $L : \mathcal{A} \to \mathcal{A}$ is said to be a *Lie derivation* if $L([X, Y]) = [L(X), Y]+[X, L(Y)]$ for all $X, Y \in \mathcal{A}$. A Lie triple derivation is an $\mathcal{R}$-linear mapping $L : \mathcal{A} \to \mathcal{A}$ which satisfies $L([[X, Y], Z]) = [[L(X), Y], Z]+[[X, L(Y)], Z]+[[X, Y], L(Z)]$ for all $X, Y, Z \in \mathcal{A}$. It can be easily seen that every derivation is a Lie derivation, and every Lie derivation is a Lie triple derivation. However, the converse statement is not true in general. Given the consideration of Lie derivations and Lie triple derivations, we can further extend them in a more general way. Suppose that $n \geq 2$ is a fixed positive integer. Let us consider the following sequence of polynomials:

$$\begin{aligned}
p_1(X_1) &\overset{\text{set}}{=} X_1, \\
p_2(X_1, X_2) &\overset{\text{set}}{=} [p_1(X_1), X_2] = [X_1, X_2], \\
\vdots \qquad & \qquad \qquad \vdots \ , \\
p_n(X_1, X_2, \cdots, X_n) &\overset{\text{set}}{=} [p_{n-1}(X_1, X_2, \cdots, X_{n-1}), X_n].
\end{aligned}$$

The polynomial $p_n(X_1, X_2, \cdots, X_n)$ is called a $(n-1)$-*commutator* $(n \geq 2)$. A$\mathcal{R}$-linear mapping $L : \mathcal{A} \longrightarrow \mathcal{A}$ is called a *Lie n-derivation* if

$$L(p_n(X_1, X_2, \cdots, X_n)) = \sum_{k=1}^{n} p_n(X_1, \cdots, X_{k-1}, L(X_k), X_{k+1}, \cdots, X_n)$$

for all $X_1, X_2, \cdots, X_n \in \mathcal{A}$. Obviously, a Lie derivation is a Lie 2-derivation and a Lie triple derivation is a Lie 3-derivation. Lie 2-derivations, Lie 3-derivations and Lie $n$-derivations are collectively referred to as *Lie-type derivations. Lie-type derivations* in different background are extensively studies by many authors (see e.g. [1, 2, 3, 4] and references therein). In the present talk, we give characterizations of Lie type derivations in the setting of triangular and generalized matrix algebras.

## keywords

derivation; Lie derivation; center. .

## References


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